Projected Linear Response Theory for Charge-Exchange Excitations and Double Beta Decay

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Abstract

The projected random phase approximation (PRPA) for charge-exchange excitations is derived from the time-dependent variational principle. Explicit results for the unperturbed energies (including the self-energy corrections), the PRPA matrices, and the transition matrix elements are presented. The effect of the projection procedure on the two-neutrino $\beta\beta$ decay in ^{76}Ge is briefly discussed.

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In a recent work [1] simple formulae have been designed for the $0^+ \to 0^+$ double beta decay ($\beta\beta$) matrix elements within the quasiparticle random phase approximation (QRPA). In particular, it has been shown that the exact calculations for the 2ν amplitude, when evaluated with a zero-range force, can be nicely fitted by a $(1,1)$-Padé approximant of the form

$$\mathcal{M}_{2\nu} \cong \mathcal{M}_{2\nu}(t=0)\frac{1-t/t_0}{1-t/t_1}, \tag{1}$$

where $t = v_t^{pp}/v_s^{pair}$ is the ratio between the spin-triplet strength in the particle-particle channel and the spin-singlet strength in the pairing channel, and t_0 and t_1 denote, respectively, the zero and the pole of $\mathcal{M}_{2\nu}$ and are treated as free parameters. In the same work it has been suggested that this result is of general validity and that any alteration of the nuclear hamiltonian or the configuration space cannot lead to a different functional dependence. [1] Thus, in order to modify the theoretical predictions for the $\beta\beta$ moments in a qualitatively way, the QRPA itself has to be altered. The simplest amendment is to set up a particle number projection. This has been done by Faessler and collaborators in several recent papers [2]. They have also discussed the outcomes of the PRPA on both simple and double beta decays. We present here a PRPA formalism that differs in several aspects from the one reported by them.

To evade the disadvantages inherent in the non-conservation of particle number, besides the usual proton quasiparticle transformation

$$\mathbf{a}_\mathsf{p} = \left(u_p\mathbf{c}_\mathsf{p} - v_p\mathbf{c}_\mathsf{\bar{p}}^\dagger\right) \quad ; \quad \mathbf{a}_\mathsf{p}^\dagger = \left(u_p\mathbf{c}_\mathsf{p}^\dagger - v_p\mathbf{c}_\mathsf{\bar{p}}\right),$$

it is useful to introduce the following *canonical* transformation [2]

$$\mathbf{d}_\mathsf{p} = \sqrt{\sigma_p}\left(u_p\mathbf{c}_\mathsf{p} - v_p z_\mathsf{p}\mathbf{c}_\mathsf{\bar{p}}^\dagger\right) ; \mathbf{d}_\mathsf{p}^\star = \sqrt{\sigma_p}\left(u_p\mathbf{c}_\mathsf{p}^\dagger - v_p z_\mathsf{p}\mathbf{c}_\mathsf{\bar{p}}\right) ; \sigma_p^{-1} = u_p^2 + z_\mathsf{p}^2 v_p^2, \tag{2}$$

and likewise for neutrons. Here $\mathbf{c}_\mathsf{p}^\dagger$ ($\mathbf{c}_\mathsf{p}$), with $\mathsf{p} \equiv p, m_p$, $p \equiv n_p \, \ell_p \, j_p$ and $m_p \equiv m_{j_p}$, are the single particle creation (annihilation) operators for protons, and $\mathbf{c}_\mathsf{\bar{p}} = (-1)^{p+m_p}\mathbf{c}_{p,-m_p}$. The

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[1]When the renormalization coupling constant g^{pp} is used [2] an analogous expression to the eq. (1) is valid (with g^{pp}'s for t's).

[2] Note that the operation symbolized by star ($\star$) does not affect the complex variable z_p.

vacuums for the proton operators $\mathbf{d_p}$ is the generating function [3, 4]

$$|z_{\mathrm{p}}\rangle = z_{\mathrm{p}}^{\hat{Z}/2}|BCS;\mathrm{p}\rangle = \prod_{\mathbf{p}>0}\left(u_p + z_{\mathrm{p}}v_p\mathbf{c}_{\mathbf{p}}^{\dagger}\mathbf{c}_{\bar{\mathbf{p}}}^{\dagger}\right)|p\rangle,$$

where $\hat{Z}$ is the particle number operator for protons, and $|p\rangle$ and $|BCS; p\rangle$ represent, respectively, the particle and the BCS vacuum for protons. The Bogoljubov transformation (2) has been introduced in the past by Ottaviani and Savoia [5] to perform a number projection in spaces of two or more BCS quasiparticles. Later, their method has been used by several authors (e.g., ref. [6]).

First, we derive the following relations for the matrix elements of the model hamiltonian H:

$$\langle BCS|H\hat{P}_0|BCS\rangle = \mathcal{T}\langle z|H|z\rangle, \tag{3}$$

$$\langle BCS|[\mathbf{A}(pn\bar{J})H\hat{P}_{\mu}\mathbf{A}^{\dagger}(p'n'J)]^0|BCS\rangle = \mathcal{T}\langle z|[\mathbf{D}_{\mu}(pn\bar{J})H\mathbf{D}_{\mu}^{\star}(p'n'J)]^0|z\rangle, \tag{4}$$

where $|BCS\rangle \equiv |BCS,p\rangle|BCS,n\rangle$ and $|z\rangle \equiv |z_{\mathrm{p}}\rangle|z_{\mathrm{n}}\rangle$. The projection operators are: $\hat{P}_0 = \hat{P}_Z\hat{P}_N$, $\hat{P}_{+1} = \hat{P}_{Z+1}\hat{P}_{N-1}$, and $\hat{P}_{-1} = \hat{P}_{Z-1}\hat{P}_{N+1}$, with $\hat{P}_{\mathcal{N}}$ being the operator for projecting states of good particle number $\mathcal{N}$ [7]. The transformation

$$\mathcal{T} = (\frac{1}{2\pi i})^2 \oint \frac{dz_{\mathrm{p}}}{z_{\mathrm{p}}^{Z+1}} \oint \frac{dz_{\mathrm{n}}}{z_{\mathrm{n}}^{N+1}},$$

is a contour integration including the origin, and

$$\mathbf{A}^{\dagger}(pnJ) = [\mathbf{a}_{\mathbf{p}}^{\dagger}\mathbf{a}_{\mathbf{n}}^{\dagger}]^J; \ \ \mathbf{D}_{\mu}^{\star}(pnJ) = \sqrt{\sigma_p\sigma_n}z_{\mathrm{p}}^{(1-\mu)/2}z_{\mathrm{n}}^{(1+\mu)/2}[\mathbf{d}_{\mathbf{p}}^{\star}\mathbf{d}_{\mathbf{n}}^{\star}]^J,$$

are the proton-neutron excitation operators in the spaces of the operators $\mathbf{a}$ and $\mathbf{d}$, respectively.

For the norms, we get

$$\langle BCS|\hat{P}_0|BCS\rangle \equiv N_0 = \mathcal{T}\mathsf{N}_0, \tag{5}$$

and

$$\hat{J}^{-1}\langle BCS|[\mathbf{A}(pn;\bar{J})\hat{P}_{\mu}\mathbf{A}^{\dagger}(pnJ)]^0|BCS\rangle \equiv N_{\mu}(pn) = \mathcal{T}\mathsf{N}_{\mu}(pn), \tag{6}$$

2

where

$$\mathsf{N}_0 = \langle z|z\rangle; \quad \mathsf{N}_\mu(pn) = z_{\mathrm{p}}^{1-\mu} z_{\mathrm{n}}^{1+\mu} \sigma_p \sigma_n \langle z|z\rangle.$$

The one-body charge-exchange operators

$$\mathcal{O}_{+1}(J) = \sum_{pn} \langle \mathsf{p}|\mathrm{O}(J)|\mathsf{n}\rangle \mathsf{c}_{\mathsf{p}}^\dagger \mathsf{c}_{\mathsf{n}}; \quad \mathcal{O}_{-1}(J) = \sum_{np} \langle \mathsf{n}|\mathrm{O}(J)|\mathsf{p}\rangle \mathsf{c}_{\mathsf{n}}^\dagger \mathsf{c}_{\mathsf{p}},$$

when expressed in terms of the $\mathbf{d}$ operators, and after neglecting the scattering terms that never contribute within a two quasiparticle space, read

$$\mathcal{O}_\mu(J) \doteq \sum_{pn} [\Lambda_\mu^0(pnJ)\mathbf{D}_\mu^\star(pnJ) + \Lambda_{-\mu}^0(pnJ)\mathbf{D}_{-\mu}(pn\bar{J})],$$

with

$$\Lambda_\mu^0(pnJ) = -\hat{J}^{-1}\langle p||\mathrm{O}(J)||n\rangle \left\{ \begin{array}{l} u_p v_n \ \text{ for } \mu = 1 \ , \\ u_n v_p \ \text{ for } \mu = -1 \ . \end{array} \right.$$

For their matrix elements we get

$$\langle BCS|[\mathbf{A}(pn\bar{J})\hat{P}_\mu \mathcal{O}_{\mu'}(J)]^0 \hat{P}_0|BCS\rangle \equiv O_\mu(pnJ)\delta_{\mu,\mu'} = \mathcal{T}\mathsf{O}_\mu(pnJ)\delta_{\mu,\mu'}, \tag{7}$$

with

$$\mathsf{O}_\mu(pnJ) = \langle z|[\mathbf{D}_\mu(pnJ)\mathcal{O}_\mu(J)]^0|z\rangle = \mathsf{N}_\mu(pn)\Lambda_\mu^0(pnJ).$$

Relations (3), (4), (5) and (6) reveal that, at the level of the Tamm-Dancoff approximation (TDA), there is a one to one correspondence between the energy spectrum of H within the projected basis $\hat{P}_\mu \mathbf{A}^\dagger(pnJ)|BCS\rangle$ (relative to the ground state $\hat{P}_0|BCS\rangle$, with energy $E_0 = \langle BCS|H\hat{P}_0|BCS\rangle/N_0$), and that within the basis $\mathbf{D}_\mu^\star(pnJ)|z\rangle$ (relative to the ground state $|z\rangle$, with energy $\mathsf{E}_0 = \langle z|H|z\rangle/\mathsf{N}_0$). As it is seen from (7) an analogous correspondence also stands for the transition matrix elements $O_\mu(pnJ)$ and $\mathsf{O}_\mu(pnJ)$. Thus within the TDA our method consists in the following: *after evaluating the matrix elements of H and $\mathcal{O}_{\pm1}(J)$ within the d-space $(< D >)$, the corresponding projected matrix elements $(< P >)$ are obtained from the transformation: $< \mathrm{P} >= \mathcal{T} < \mathrm{D} >$.*

3

Next, the above method is extended to the RPA by following the linear response procedure developed by Lane and Martorell [8]. We introduce the Slater determinant

$$|\phi(z)\rangle = \prod_{pnJ\mu} (1 + \chi_\mu(pnJ)\mathbf{D}_\mu^\star(pnJ))|z\rangle, \tag{8}$$

and if the quantities χ are assumed to be small, then, to second order

$$\langle\phi(z)|\phi(z)\rangle = \mathsf{N}_0 + \sum_{pnJ\mu} \left[|\chi_\mu(pnJ)|^2 \, \mathsf{N}_\mu(pn) + |\chi_{-\mu}(pnJ)|^2 \, \mathsf{N}_{-\mu}(pn) \right]. \tag{9}$$

The response of the system to an external time-dependent field

$$\hat{\mathcal{O}}_\mu = \mathcal{O}_\mu e^{-i\omega t} + \mathcal{O}_\mu^\dagger e^{i\omega t},$$

is derived from the time dependent variational principle, which for the optimum Slater determinant solution of the perturbed problem gives

$$\langle\delta\phi(z)|H - \mathsf{E}_0 + \hat{\mathcal{O}}_\mu - i\hbar\frac{\partial}{\partial t}|\phi(z)\rangle = 0.$$

If $\chi_\mu(pnJ)$ are varied, keeping only the linear terms and assuming $\langle z|\hat{\mathcal{O}}_\mu|z\rangle = 0$, one gets

$$\mathsf{A}_\mu\chi_\mu + \mathsf{B}\chi_{-\mu}^* + \mathsf{O}_\mu e^{-i\omega t} - i\hbar\dot{\chi}_\mu\mathsf{N}_\mu = 0,$$
$$\mathsf{B}_\mu^*\chi_\mu + \mathsf{A}_{-\mu}^*\chi_{-\mu}^* + \mathsf{O}_{-\mu}^* e^{-i\omega t} - i\hbar\dot{\chi}_{-\mu}^*\mathsf{N}_{-\mu}^* = 0,$$

where the submatrices are given by

$$\mathsf{A}_\mu(pn, p'n'J) = \hat{J}^{-1}\langle z|[\mathbf{D}_\mu(pn\bar{J})(H - \mathsf{E}_0\mathbf{1})\mathbf{D}_\mu^\star(p'n'J)]^0|z\rangle$$
$$\mathsf{B}(pn, p'n'J) = \hat{J}^{-1}\langle z|[\mathbf{D}_{-\mu}(pn\bar{J})\mathbf{D}_\mu(p'n'J)H]^0|z\rangle.$$

Attempting a solution of the form

$$\chi_\mu = X_\mu e^{-i\omega t}; \quad \chi_{-\mu} = Y_\mu^* e^{i\omega t}, \tag{10}$$

and performing the transformation $(A_\mu, B, N_\mu, O_\mu) = \mathcal{T}(\mathsf{A}_\mu, \mathsf{B}, \mathsf{N}_\mu, \mathsf{O}_\mu)$, we obtain

$$\begin{pmatrix} A_\mu - \hbar\omega N_\mu\mathbf{1} & B \\ B^* & A_{-\mu}^* + \hbar\omega N_{-\mu}^*\mathbf{1} \end{pmatrix} \begin{pmatrix} X_\mu \\ Y_\mu \end{pmatrix} = -\begin{pmatrix} O_\mu \\ O_{-\mu}^* \end{pmatrix}. \tag{11}$$

4

The corresponding RPA eigenvalue problem is:

$$\begin{pmatrix} A_\mu & B \\ B^* & A_{-\mu}^* \end{pmatrix} \begin{pmatrix} X_\mu(\alpha) \\ Y_\mu(\alpha) \end{pmatrix} = E_\mu(\alpha) \begin{pmatrix} N_\mu & 0 \\ 0 & -N_{-\mu}^* \end{pmatrix} \begin{pmatrix} X_\mu(\alpha) \\ Y_\mu(\alpha) \end{pmatrix},\tag{12}$$

with the eigenvectors normalized such as to obey the orthogonal relation:

$$X_\mu^\dagger(\alpha)N_\mu X_\mu(\beta) - Y_\mu^\dagger(\alpha)N_{-\mu}Y_\mu(\beta) = \begin{cases} \delta_{\alpha\beta} & \text{for } E_\mu(\alpha) > 0 , \\ -\delta_{\alpha\beta} & \text{for } E_\mu(\alpha) < 0 , \end{cases}$$

while the closure relations are:

$$\sum_{E_\mu(\alpha)>0} [X_\mu(\alpha)X_\mu^\dagger(\alpha) - Y_{-\mu}^*(\alpha)\tilde{Y}_{-\mu}(\alpha)]N_\mu = \mathbf{1},$$

$$\sum_{E_\mu(\alpha)>0} [X_\mu^*(\alpha)\tilde{Y}_\mu(\alpha) - Y_{-\mu}(\alpha)X_{-\mu}^\dagger(\alpha)]N_{-\mu}^* = 0.$$

We interpret $E_\mu(\alpha J)$ as the PRPA approximation for the exact excitation energies from the ground state in the nucleus $(Z,\ N)$ to the states $|\alpha J; \mu\rangle$ in the neighboring odd-odd nuclei: $E_\mu(\alpha J) = \langle \alpha J; \mu|H|\alpha J; \mu\rangle_{PRPA} - \langle 0|H|0\rangle_{PRPA}$. Note that in contrast with the usual charge-conserving RPA problem, the eigenvalues do not occur in pairs $\pm E_\mu(\alpha J)$. Besides, both positive and the negative solutions are physically meaningful. For $\mu = \pm 1$ the positive solutions describe excitations in the $(Z\pm 1, N\mp 1)$ nuclei, while the negative energy solutions represent the de-excitations in the $(Z\pm 1, N\mp 1)$ nuclei, and the positive energy excitations in the $(Z\mp 1, N\pm 1)$ nuclei as well. Thus only one RPA equation has to be solved for the evaluation of single β^- and β^+ transitions. This is a well known feature of the charge-exchange modes [8, 9, 10].

We can express the solution of eq. (11) in terms of the eigenvectors of (12),

$$\begin{pmatrix} X_\mu \\ Y_\mu \end{pmatrix} = -\sum_\alpha sgn(E_\mu(\alpha)) \begin{pmatrix} X_\mu(\alpha) \\ Y_\mu(\alpha) \end{pmatrix} \frac{1}{E_\mu(\alpha) - \hbar\omega}(X_\mu^\dagger(\alpha)\ Y_\mu^\dagger(\alpha)) \begin{pmatrix} O_\mu \\ O_{-\mu}^* \end{pmatrix},$$

and hence from (8), (9) and (10) we derive the response function [8],

$$R(\hbar\omega) \equiv -\frac{1}{2}\frac{\mathcal{T}\langle\phi(z)|\hat{\mathcal{O}}_\mu|\phi(z)\rangle}{\mathcal{T}\langle\phi(z)|\phi(z)\rangle} = \sum_{\alpha J} sgn(E_\mu(\alpha J))\frac{|\Lambda_\mu(\alpha J)|^2}{E_\mu(\alpha J) - \hbar\omega},$$

where

$$\Lambda_\mu(\alpha J) = N_0^{-1/2} \sum_{pn} [\Lambda_\mu^0(pnJ)N_\mu(pn)X_\mu^*(pn;\alpha J) + \Lambda_{-\mu}^0(pnJ)N_{-\mu}(pn)Y_\mu^*(pn;\alpha J)],$$

is the PRPA approximation to the matrix element of $\mathcal{O}_\mu(J)$ between the exact counterparts of $|\alpha J;\mu\rangle$ and the ground state, i.e., $\Lambda_\mu(\alpha J) = \langle \alpha J;\mu||\mathcal{O}_\mu(J)||0\rangle_{PRPA}$.

Introducing the normalized amplitudes,

$$\mathcal{X}_\mu(pn;\alpha J) = N_\mu^{1/2}(pn)X_\mu(pn;\alpha J); \quad \mathcal{Y}_\mu(pn;\alpha J) = N_{-\mu}^{1/2}(pn)Y_\mu(pn;\alpha J),$$

the PRPA equation takes the standard form

$$\begin{pmatrix} \mathcal{A}_\mu(J) & \mathcal{B}(J) \\ -\mathcal{B}^\dagger(J) & -\mathcal{A}_{-\mu}^*(J) \end{pmatrix} \begin{pmatrix} \mathcal{X}_\mu(\alpha J) \\ \mathcal{Y}_\mu(\alpha J) \end{pmatrix} = E_\mu(\alpha J) \begin{pmatrix} \mathcal{X}_\mu(\alpha J) \\ \mathcal{Y}_\mu(\alpha J) \end{pmatrix},$$

with

$$\mathcal{A}_\mu(pn, p'n'; J) = \frac{A_\mu(pn, p'n'; J)}{\sqrt{N_\mu(pn)N_\mu(p'n')}},$$

$$\mathcal{B}(pn, p'n'; J) = \frac{B(pn, p'n'; J)}{\sqrt{N_\mu(pn)N_{-\mu}(p'n')}}.$$

The orthogonality and closure relations are, respectively,

$$\mathcal{X}_\mu^\dagger(\alpha)\mathcal{X}_\mu(\beta) - \mathcal{Y}_\mu^\dagger(\alpha)\mathcal{Y}_\mu(\beta) = \begin{cases} \delta_{\alpha\beta} & \text{for } E_\mu(\alpha) > 0, \\ -\delta_{\alpha\beta} & \text{for } E_\mu(\alpha) < 0. \end{cases}$$

and:

$$\sum_{E_\mu(\alpha)>0} [\mathcal{X}_\mu(\alpha)\mathcal{X}_\mu^\dagger(\alpha) - \mathcal{Y}_{-\mu}^*(\alpha)\tilde{\mathcal{Y}}_{-\mu}(\alpha)] = \mathbf{1},$$

$$\sum_{E_\mu(\alpha)>0} [\mathcal{X}_\mu^*(\alpha)\tilde{\mathcal{Y}}_\mu(\alpha) - \mathcal{Y}_{-\mu}(\alpha)\mathcal{X}_{-\mu}^\dagger(\alpha)] = 0,$$

while the transition amplitudes read

$$\Lambda_\mu(\alpha J) = N_0^{-1/2} \sum_{pn} [\Lambda_\mu^0(pnJ)N_\mu^{1/2}(pn)\mathcal{X}_\mu^*(pn;\alpha J) + \Lambda_{-\mu}^0(pnJ)N_{-\mu}^{1/2}(pn)\mathcal{Y}_\mu^*(pn;\alpha J)], \quad (13)$$

and for $J^\pi = 0^+$ and 1^+ satisfy the sum-rule [8]

$$\sum_{E_{\mu=1}(\alpha J)>0} |\Lambda_\mu(\alpha J)|^2 - \sum_{E_{\mu=-1}(\alpha J)>0} |\Lambda_{-\mu}(\alpha J)|^2 = (2J+1)(N-Z).$$

The explicit results for the projected RPA matrices are:

$$A_\mu(pn, p'n'; J) = \epsilon^{Z-1+\mu, N-1-\mu}(pn)\delta_{pn,p'n'} + R_{22}^{Z-1+\mu, N-1-\mu}(pnp'n'; J),$$
$$B(pn, p'n'; J) = R_{40}^{Z,N}(pn, p'n'; J),$$

where

$$\begin{aligned}
\epsilon^{k,k'}(pn) &= [R_0^k(p) + R_{11}^k(pp)]I^{k'}(n) + [R_0^{k'}(n) + R_{11}^{k'}(nn)]I^k(p) \\
&+ R_0^{k,k'}(pn) + R_{11}^{k,k'}(pn) - E_0 I^k(p) I^{k'}(n),
\end{aligned} \tag{14}$$

are the unperturbed energies, and

$$E_0 = \frac{R_0^Z}{I^Z} + \frac{R_0^N}{I^N} + \frac{R_0^{Z,N}}{I^Z I^N}. \tag{15}$$

is the ground state energy. The norms are:

$$N_0 = I^Z I^N; \quad N_\mu(pn) = I^{Z-1+\mu}(p)I^{N-1-\mu}(n),$$

and the remaining quantities are defined as follows:

$$\begin{aligned}
R_0^k(p_1 \cdot\cdot) &= \sum_p \hat{j}_p^2 v_p^2 e_p I^{k-2}(pp_1 \cdot\cdot) \\
&+ \frac{1}{4}\sum_{pp'} \hat{j}_p \hat{j}_{p'}[2v_p^2 v_{p'}^2 \mathrm{f}(pp')I^{k-4}(pp'p_1 \cdot\cdot) + u_p v_p u_{p'} v_{p'} \mathrm{g}(pp')I^{k-2}(pp'p_1 \cdot\cdot)],
\end{aligned}$$

$$\begin{aligned}
R_{11}^k(p_1 p_2 \cdot\cdot) &= e_{p_1}[u_{p_1}^2 I^k(p_1 p_2 \cdot\cdot) - v_{p_1}^2 I^{k-2}(p_1 p_2 \cdot\cdot)] \\
&+ \hat{j}_{p_1}^{-1}\sum_p \hat{j}_p\{v_p^2 \mathrm{f}(pp_1)[u_{p_1}^2 I^{k-2}(pp_1 p_2 \cdot\cdot) - v_{p_1}^2 I^{k-4}(pp_1 \cdot\cdot)] \\
&- u_p v_p u_{p_1} v_{p_1} \mathrm{g}(pp_1)I^{k-2}(pp_1 p_2 \cdot\cdot)\},
\end{aligned}$$

7

$$R_0^{k,k'}(p_1 \cdots, n_1 \cdots) = \sum_{pn} \hat{j}_p \hat{j}_n v_p^2 v_n^2 \mathrm{f}(pn) I^{k-2}(pp_1 \cdots) I^{k'-2}(nn_1 \cdots)),$$

$$
\begin{aligned}
R_{11}^{k,k'}(pn) &= \hat{j}_p^{-1}[u_p^2 I^k(pp) - v_p^2 I^{k-2}(pp)] \sum_{n'} \hat{j}_{n'} v_{n'}^2 \mathrm{f}(pn') I^{k'-2}(nn') \\
&+ \hat{j}_n^{-1}[u_n^2 I^{k'}(nn) - v_n^2 I^{k'-2}(nn)] \sum_{p'} \hat{j}_{p'} v_{p'}^2 \mathrm{f}(np') I^{k-2}(pp'),
\end{aligned}
$$

$$
\begin{aligned}
R_{22}^{k,k'}(pn, p'n'; J) &= [u_p v_n u_{p'} v_{n'} I^k(pp') I^{k'-2}(nn') + v_p u_n v_{p'} u_{n'} I^{k-2}(pp') I^{k'}(nn')] \mathrm{F}(pn, p'n'; J) \\
&+ [u_p u_n u_{p'} u_{n'} I^k(pp') I^{k'}(nn') + v_p v_n v_{p'} v_{n'} I^{k-2}(pp') I^{k'-2}(nn')] \mathrm{G}(pn, p'n'; J) \\
R_{40}^{Z,N}(pn, p'n'; J) &= I^{Z-2}(pp') I^{N-2}(nn') \left[(v_p u_n u_{p'} v_{n'} + u_p v_n v_{p'} u_{n'}) \mathrm{F}(pn, p'n'; J) \right. \\
&- \left. (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \mathrm{G}(pn, p'n'; J) \right].
\end{aligned}
$$

where e_p are the proton single particle energies (s.p.e.), $\mathrm{f}(pp') \equiv \mathrm{F}(pp, p'p'; J = 0)$, $\mathrm{g}(pp') \equiv \mathrm{G}(pp, p'p'; J = 0)$, $\mathrm{f}(np) \equiv \mathrm{F}(nn, pp; J = 0)$ and

$$I^k(p_1 p_2 \cdots p_n) = \frac{1}{2\pi i} \oint \frac{dz_{\mathrm{p}}}{z_{\mathrm{p}}^{k+1}} \sigma_{p_1} \sigma_{p_2} \cdots \sigma_{p_n} \prod_p (u_p^2 + z_{\mathrm{p}}^2 v_p^2)^{\hat{j}_p^2/2}.$$

The starting point of a self-consistent numerical work is the minimization of E_0 with regard to the parameters u and v. This leads to a set of coupled equations [4, 5]:

$$2\hat{e}_p u_p v_p - \Delta_p(u_p^2 - v_p^2) = 0, \tag{16}$$

where the quantities $\hat{e}_p$ and Δ_p are defined as follows:

$$\Delta_p = -\frac{1}{2}\hat{j}_p^{-1} \sum_{p'} \hat{j}_{p'} u_{p'} v_{p'} \mathrm{g}(pp') I^{Z-4}(p),$$

$$
\begin{aligned}
\hat{e}_p &= \bar{e}_p I^{Z-2}(p) + \hat{j}_p^{-1} \sum_{p'} v_{p'}^2 \mathrm{f}(pp') I^{Z-4}(pp') \\
&+ \frac{1}{2}\hat{j}_p^{-2} \sum_{p'} \hat{j}_{p'}^2 v_{p'}^2 \bar{e}_{p'} \left[\nu_p(p') D^{Z-4}(pp') - \nu_p D^{Z-2}(p') \frac{I^{Z-2}(p)}{I^Z} \right] \\
&+ \frac{1}{8}\hat{j}_p^{-2} \sum_{p'p''} \hat{j}_{p'} \hat{j}_{p''} \left\{ 2v_{p'}^2 v_{p''}^2 \mathrm{f}(p'p'') \left[\nu_p(p'p'') D^{Z-6}(pp'p'') - \nu_p D^{Z-2}(p) \frac{I^{Z-4}(p'p'')}{I^Z} \right] \right. \\
&+ \left. u_{p'} v_{p'} u_{p''} v_{p''} \mathrm{g}(p'p'') \left[\nu_p(p'p'') D^{Z-4}(pp'p'') - \nu_p D^{Z-2}(p) \frac{I^{Z-2}(p'p'')}{I^Z} \right] \right\}
\end{aligned}
$$

with

$$\bar{e}_p = e_p + \hat{j}_p^{-1} \sum_n \hat{j}_n v_n^2 \mathrm{f}(pn) I^{N-2}(n)/I^N,$$

$$\nu_p(p' \cdots) = \hat{j}_p^2 - 2(\delta_{pp'} + \cdots),$$

$$D^k(p \cdots) = I^k(p \cdots) - I^{k+2}(p \cdots).$$

To inquire into the effect of the number projection method we have done numerical calculations for several $2\nu\beta\beta$ emitting nuclei, following the procedure outlined in ref. [12]. This implies the employment of the s.p.e. from the neighboring odd-odd nuclei, and therefore all self-energy corrections in eqs. (14), (15) and (16), for both like and unlike particles, have been neglected. Besides, and for the sake of comparison with results presented in ref. [2], the $0^+ \to 0^+$ 2ν transition amplitude has been approximated as

$$\mathcal{M}_{2\nu} = \frac{1}{2} \sum_{\alpha,\alpha'} \left(\frac{1}{E_{\mu=1}(\alpha J = 1) + \Delta} + \frac{1}{\overline{E}_{\mu=-1}(\alpha' J = 1) - \Delta} \right)$$
$$\times \Lambda_{\mu=1}(\alpha J = 1)\langle \alpha J = 1; \mu = 1 | \overline{\alpha' J = 1; \mu = -1} \rangle \overline{\Lambda}_{\mu=-1}(\alpha' J = 1), \qquad (17)$$

where unbarred and barred quantities indicate that the quasiparticles and excitations are defined with respect to the initial (Z, N) nucleus, and final $(Z+2, N-2)$ nucleus, respectively, and $\Delta = \left(E_0 - \overline{E}_0 \right)/2$.

As an example, the PQRPA and QRPA calculations for the $^{76}Ge \to {}^{76}Se$ transition are displayed in fig. 1. Also, in the same figure it is shown the result of a hybrid model calculation, in which the PQRPA is used, but with the BCS parameters u and v employed within the QRPA. It can be observed that all the three results are qualitative similar, in the sense that the $\beta\beta$ moments always vary rather abruptly in the physically relevant interval $t_0 \gtrsim t \gtrsim t_1$, and are well represented by eq. (1); the corresponding parameters t_0 and t_1 that fit the exact results, together with $\mathcal{M}_{2\nu}(t = 0)$, are listed in table 1. The single mode model for the $\beta\beta$ decay [1, 13] suggests that the dominant effect of the projection method is to insert, into the numerator and the denominator of eq. (1), terms of the order of $\left(\Omega_p^{-1} + \Omega_n^{-1} \right)$, where $2\Omega_p$ and $2\Omega_n$ are, respectively, the degeneracy of the proton and neutron levels.

We end the present letter by mentioning the main differences between the PRPA formalism reported in ref. [2] and the one presented here:

a) They solve two different RPA equations for the $(Z+1, N-1)$ and $(Z-1, N+1)$ nuclei. Contrarily, we only deal with one RPA problem, in which the β^+ spectrum is viewed as an extension of the β^- spectrum to negative energies [8, 9, 10].

b) Their matrices A and B are defined through the method of commutation relations. Yet, we feel that this technique cannot be applied with the projected procedure, as it automatically excludes the R_0 terms in the expression (14) for the unperturbed energies. But these terms in no way cancel among themselves [5, 6], and thus, our unperturbed energies are necessarily distinct from those obtained in ref. [2]. (It is worth noting that the commutation method has neither been used in the derivation of the PRPA for like particles [11].)

c) Their result for the matrix B also disagrees with our result and we do not find any explanation for such a discrepancy. In our formalism the ground state correlations always occur in the (Z, N) nucleus.

d) We do not have to deal with spurious components in the wave functions of the intermediate double-odd nucleus as it apparently happens in ref. [2], and therefore $A_{\mu=1}$, $A_{\mu=-1}$ and B are square matrices of dimension equal to the number of proton-neutron states with a given spin and parity.

e) The matrix element for the one-body transition operator, given by eq. (13), has not been explicitly shown in their previous works.

In summary, the QRPA and PQRPA yield qualitatively similar results, and, in spite of above mentioned differences between the formalism employed by the Tübingen group and ours, the numerical results that we get for the moments $\mathcal{M}_{2\nu}$ are not in essence distinct from those obtained in ref. [2].

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Figure Captions

Figure 1: Calculated double beta decay matrix elements $\mathcal{M}_{2\nu}$ (in units of $[MeV]^{-1}$) for ^{76}Ge, as a function of the particle-particle $S = 1, T = 0$ coupling constant t. Solid and dotted curves correspond to the projected (PQRPA) and unprojected (QRPA) results, respectively. For comparison a hybrid model result, in which the PQRPA is used with parameters u and v determined as in the QRPA, is also presented (dashed curve).

Tables

Table 1: The coefficients $\mathcal{M}_{2\nu}(t = 0)$, t_0 and t_1 for ^{76}Ge in the parametrization of 2ν moments by eq. (1). The exact curves, shown in fig. 1, are not distinguished visually from the fitted ones with the parameters listed here.

	QRPA	PQRPA	PQRPA (hybrid)
$-\mathcal{M}_{2\nu}(t = 0)$	0.385	0.370	0.297
t_0	1.170	1.038	0.950
t_1	1.591	1.447	1.421

This figure "fig1-1.png" is available in "png" format from:

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